

LATTICE-TOPOLOGICAL PROPERTIES OF WALLMAN SPACES OF σ -SMOOTH ZERO-ONE VALUED MEASURES

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RÉSUMÉ. On s'intéresse aux sous-ensembles de $I(\mathcal{L})$ composés de mesures σ -lisses, fortement σ -lisses ou σ -lisses et régulières. L'espace de Wallman qui résulte d'un tel sous-ensemble peut être normal, régulier, replet, complet ou compact, et la topologie engendrée possède des propriétés comme celle d'être T_2 et Lindelöf.

ABSTRACT. This paper is concerned with various subsets of the set $I(\mathcal{L})$ consisting of σ -smooth, strongly σ -smooth and regular σ -smooth measures. The resulting Wallman spaces are investigated for normality, regularity, repleteness, completeness and compactness and the topologies generated by them are discussed for various properties such as T_2 and Lindelöf.

1. Introduction. Let X be an arbitrary non empty set and $\mathcal{L}$ a lattice of subsets of X such that $\emptyset, X \in \mathcal{L}$. $\mathcal{A}(\mathcal{L})$ is the algebra generated by $\mathcal{L}$ and $I(\mathcal{L})$ denotes those non trivial, zero-one valued, finitely additive measures on $\mathcal{A}(\mathcal{L})$.

Various subsets of $I(\mathcal{L})$ are considered and certain lattices in these subsets are investigated as well as the topology of closed sets generated by them. The lattices are investigated for normality, regularity and for a variety of repleteness, completeness and compactness conditions. The topologies are similarly investigated for various properties such as T_2 and Lindelöf. Necessary and sufficient conditions for these properties to hold can effectively be given in terms of measure conditions on the original lattices.

Some investigations in these matters have been done in [2, 5, 14]. We go beyond these results and introduce new subsets of $I(\mathcal{L})$ and their lattices to investigate.

We begin with some standard notations and terminology that will be used throughout the paper. Our notations and terminology is consistent with that in the literature (see *e.g.* [1, 2, 8, 15]), and is added mainly for the reader's convenience. We summarize some known results about zero-one valued measures and filters correspondence (see [7]) and use them throughout the paper, since one could use analogous filter arguments.

We then proceed in the subsequent sections to analyze in detail the lattice-topological structure of various Wallman spaces as indicated above.

2. Background and notations. Let X be an arbitrary non empty set and $\mathcal{L}$ a lattice of subsets of X such that $\emptyset, X \in \mathcal{L}$. $\mathcal{A}(\mathcal{L})$ is the algebra generated by $\mathcal{L}$.

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Definition 1: lattice terminology. The lattice $\mathcal{L}$ is called: *disjunctive* if for $x \in X$ and $L_1 \in \mathcal{L}$ such that $x \notin L_1$ there exists $L_2 \in \mathcal{L}$ with $x \in L_2$ and $L_1 \cap L_2 = \emptyset$; *separating* (or T_1) if $x, y \in X$ and $x \neq y$ implies there exists $L \in \mathcal{L}$ such that $x \in L$, $y \notin L$; T_2 if for $x, y \in X$ and $x \neq y$ there exist $L_1, L_2 \in \mathcal{L}$ such that $x \in L'_1$, $y \in L'_2$ and $L'_1 \cap L'_2 = \emptyset$ (where the prime denotes the complement); *complement generated* if $L \in \mathcal{L}$ implies $L = \bigcap_{n=1}^{\infty} L'_n$, $L_n \in \mathcal{L}$.

Definition 2: measure terminology. $I(\mathcal{L})$ denotes those non trivial, zero-one valued, finitely additive measures on $\mathcal{A}(\mathcal{L})$.

$I_R(\mathcal{L})$ is the set of $\mathcal{L}$ -regular measures of $I(\mathcal{L})$, where $\mu \in I(\mathcal{L})$ is $\mathcal{L}$ -regular if for any $A \in \mathcal{A}(\mathcal{L})$,

$$\mu(A) = \sup\{\mu(L) \mid L \subseteq A, L \in \mathcal{L}\}.$$

$I_\sigma(\mathcal{L})$ is the set of σ -smooth measures of $I(\mathcal{L})$ on $\mathcal{L}$, where $\mu \in I(\mathcal{L})$ is σ -smooth on $\mathcal{L}$ if for all sequences $\{L_n\}$ of sets of $\mathcal{L}$ with $L_n \downarrow \emptyset$, $\mu(L_n) \rightarrow 0$. $I^\sigma(\mathcal{L})$ is the set of σ -smooth measures on $\mathcal{A}(\mathcal{L})$ of $I(\mathcal{L})$. $I_R^\sigma(\mathcal{L})$ is the set of $\mathcal{L}$ -regular measures of $I^\sigma(\mathcal{L})$. $\mathcal{P}(\mathcal{L}) = \{\pi \text{ defined on } \mathcal{L}, \text{ non trivial, monotone and } \pi(A \cap B) = \pi(A)\pi(B), A, B \in \mathcal{L}\}$ is the set of all *premeasures* on $\mathcal{L}$. $\mathcal{P}_\sigma(\mathcal{L})$ is the set of all premeasures on $\mathcal{L}$ which are σ -smooth on $\mathcal{L}$.

Definition 3: filters-measures correspondence. A filter $\mathcal{F}$ in $\mathcal{L}$ is a subset of $\mathcal{L}$ satisfying the conditions: $\emptyset \notin \mathcal{F}$; $\mathcal{F}$ is closed under finite intersections; if $A \in \mathcal{F}$, $B \in \mathcal{L}$ and $A \subset B$, then $B \in \mathcal{F}$. An ultrafilter in $\mathcal{L}$ is a maximal filter (relative to the partial order on the collection of filters in $\mathcal{L}$ given by inclusion). An $\mathcal{L}$ -filter $\mathcal{F}$ is prime if given $A, B \in \mathcal{L}$ such that $A \cup B \in \mathcal{F}$ then either $A \in \mathcal{F}$ or $B \in \mathcal{F}$.

There exists a one-to-one correspondence between:

- $\mathcal{L}$ -filters (and elements of $\mathcal{P}(\mathcal{L})$ defined by $\pi(L) = 1$ if, and only if, $L \in \mathcal{F}$;
- $\mathcal{L}$ -filters $\mathcal{F}$ with countable intersection property and elements of $\mathcal{P}_\sigma(\mathcal{L}) = \{\pi \in \mathcal{P}(\mathcal{L}) \text{ such that if } \pi(L_n) = 1 \text{ for all } n \text{ where } L_n \in \mathcal{L} \text{ then } \bigcap L_n \neq \emptyset\}$;
- Prime $\mathcal{L}$ -filters and elements of $I(\mathcal{L})$ given by: for any $\mu \in I(\mathcal{L})$ we associate the prime $\mathcal{L}$ -filter $\mathcal{F} = \{A \in \mathcal{L} \mid \mu(A) = 1\}$;
- Prime $\mathcal{L}$ -filters with countable intersection property and elements of $I_\sigma(\mathcal{L})$;
- $\mathcal{L}$ -ultrafilters and elements of $I_R(\mathcal{L})$ given by: for any $\mathcal{L}$ -ultrafilter $\mathcal{F}$ we associate the zero-one valued measure defined on $\mathcal{A}(\mathcal{L})$ by: $\mu(E) = 1$ if there exists $A \in \mathcal{F}$, $A \subset E$ and $\mu(E) = 0$ if there exists $A \in \mathcal{F}$, $A \subset E'$;
- $\mathcal{L}$ -ultrafilters with countable intersection property and elements of $I_R^\sigma(\mathcal{L})$.

For more filter-related matters, see [7].

Definition 4: lattice-topological properties. The *support* of $\mu \in I(\mathcal{L})$ is $S(\mu) = \bigcap\{L \in \mathcal{L} \mid \mu(L) = 1\}$. With this notation, we now note:

- $\mathcal{L}$ is *compact* if, and only if, $S(\mu) \neq \emptyset$ for every $\mu \in I_R(\mathcal{L})$.
- $\mathcal{L}$ is *countably compact* if, and only if, $I_R(\mathcal{L}) = I_R^\sigma(\mathcal{L})$.
- $\mathcal{L}$ is *normal* if, and only if, for each $\mu \in I(\mathcal{L})$, there exists a unique $\nu \in I_R(\mathcal{L})$ such that $\mu \leq \nu$ i.e. $\mu(L) \leq \nu(L)$ for all $L \in \mathcal{L}$.
- $\mathcal{L}$ is *regular* if, and only if, whenever $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\mu_1 \leq \mu_2$, then $S(\mu_1) = S(\mu_2)$.

- $\mathcal{L}$ is *replete* if, and only if, for any $\mu \in I_R^\sigma(\mathcal{L})$, $S(\mu) \neq \emptyset$.
- $\mathcal{L}$ is *prime-complete* if, and only if, for any $\mu \in I_\sigma(\mathcal{L})$, $S(\mu) \neq \emptyset$.
- $\mathcal{L}$ is *Lindelöf* if, and only if, for any $\pi \in \mathcal{P}_\sigma(\mathcal{L})$, $S(\pi) \neq \emptyset$.
- Finally, if μ_x is the measure concentrated at $x \in X$ then $\mu_x \in I_R(\mathcal{L})$, for all $x \in X$ if, and only if, $\mathcal{L}$ is *disjunctive*.

For further results and related matters see [10, 13, 14].

3. The spaces $I_R^\sigma(\mathcal{L})$, $I_\sigma(\mathcal{L})$ and the lattices $\mathcal{W}_\sigma(\mathcal{L})$, $\mathcal{V}_\sigma(\mathcal{L})$. Next, we briefly summarize some facts (see [3]) about the Wallman space $I_R^\sigma(\mathcal{L})$ and then proceed to consider some relations between the measures and the induced measures on the Wallman spaces. A few of the properties to be considered have been investigated in [5], we give slightly different proofs, and include some of them for completeness.

Definition 5: general Wallman spaces and Wallman topologies. The *Wallman topology* in $I_R^\sigma(\mathcal{L})$ is obtained by taking all

$$W_\sigma(L) = \{\mu \in I_R^\sigma(\mathcal{L}) \mid \mu(L) = 1\}, \quad L \in \mathcal{L}$$

as a base for the closed sets in $I_R^\sigma(\mathcal{L})$ and then $I_R^\sigma(\mathcal{L})$ is called the *general Wallman space* associated with X and $\mathcal{L}$. Assuming $\mathcal{L}$ is disjunctive, $\mathcal{W}_\sigma(\mathcal{L}) = \{W_\sigma(L) \mid L \in \mathcal{L}\}$ is a lattice in $I_R^\sigma(\mathcal{L})$, isomorphic to $\mathcal{L}$ under the map $L \rightarrow W_\sigma(L)$, $L \in \mathcal{L}$. $\mathcal{W}_\sigma(\mathcal{L})$ is replete and a base for the closed sets of the topology $t\mathcal{W}_\sigma(\mathcal{L})$, all arbitrary intersections of sets of $\mathcal{W}_\sigma(\mathcal{L})$. It is this topological space $[I_R^\sigma(\mathcal{L}), t\mathcal{W}_\sigma(\mathcal{L})]$ and lattice $\mathcal{W}_\sigma(\mathcal{L})$ which we will consider here and subsequent sections.

Analogously, we also consider $I_\sigma(\mathcal{L})$ and $\mathcal{V}_\sigma(\mathcal{L}) = \{V_\sigma(L) \mid L \in \mathcal{L}\}$ where $V_\sigma(A) = \{\mu \in I_\sigma(\mathcal{L}) \mid \mu(A) = 1\}$, $A \in \mathcal{A}(\mathcal{L})$. Note that here we do not need the assumption of disjunctiveness on $\mathcal{L}$ and that $\mathcal{V}_\sigma(\mathcal{L})$ is prime complete and a base for the closed sets $t\mathcal{V}_\sigma(\mathcal{L})$ of $I_\sigma(\mathcal{L})$. Note also that in a similar way one can construct the Wallman topological spaces $[I_R(\mathcal{L}), t\mathcal{W}(\mathcal{L})]$, (here $\mathcal{L}$ must be separating, too) and $[I(\mathcal{L}), t\mathcal{V}(\mathcal{L})]$ and in Section 6 will see yet another Wallman space.

If $A \in \mathcal{A}(\mathcal{L})$, then $W_\sigma(A) = \{\mu \in I_R^\sigma(\mathcal{L}) \mid \mu(A) = 1\}$ and the following statements are true (see [3]):

- $W_\sigma(A \cup B) = W_\sigma(A) \cup W_\sigma(B)$
- $W_\sigma(A \cap B) = W_\sigma(A) \cap W_\sigma(B)$
- $W_\sigma(A') = W_\sigma(A)'$
- $A \supset B$ if, and only if, $W_\sigma(A) \supset W_\sigma(B)$
- $\mathcal{A}(\mathcal{W}_\sigma(\mathcal{L})) = \mathcal{W}_\sigma(\mathcal{A}(\mathcal{L}))$.

Definition 6: the induced measure. Let $\mu \in I_R^\sigma(\mathcal{L})$ and consider the *induced measure* $\underline{\mu} \in I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$, defined by

$$\underline{\mu}(W_\sigma(A)) = \mu(A), \quad A \in \mathcal{A}(\mathcal{L}).$$

The map $\mu \rightarrow \underline{\mu}$ is a bijection between $I_R^\sigma(\mathcal{L})$ and $I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$.

Theorem 3.1. (a) Consider $I_R^\sigma(\mathcal{L})$ and $\mathcal{W}_\sigma(\mathcal{L})$ with $\mathcal{L}$ disjunctive. $\mathcal{W}_\sigma(\mathcal{L})$ is regular if, and only if, for all $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\nu \in I_R^\sigma(\mathcal{L})$, if $\mu_1 \leq \mu_2, \nu(\mathcal{L})$ then $\mu_2 \leq \nu(\mathcal{L})$.

(b) Consider the space $[I_R^\sigma(\mathcal{L}), t\mathcal{W}_\sigma(\mathcal{L})]$ with $\mathcal{L}$ disjunctive. The space is T_2 if, and only if, for $\mu \in I(\mathcal{L})$ and $\mu \leq \nu_1, \nu_2(\mathcal{L})$, where $\nu_1, \nu_2 \in I_R^\sigma(\mathcal{L})$ it follows that $\nu_1 = \nu_2$.

(c) Consider $I_\sigma(\mathcal{L})$ and $\mathcal{V}_\sigma(\mathcal{L})$. $\mathcal{V}_\sigma(\mathcal{L})$ is regular if, and only if, for all $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\nu \in I_\sigma(\mathcal{L})$, if $\mu_1 \leq \mu_2(\mathcal{L})$ and $\mu_1 \leq \nu(\mathcal{L})$ then $\mu_2 \leq \nu(\mathcal{L})$.

(d) Consider the space $[I_\sigma(\mathcal{L}), t\mathcal{V}_\sigma(\mathcal{L})]$. The space is T_2 if, and only if, for $\mu \in I(\mathcal{L})$ and $\mu \leq \nu_1, \nu_2(\mathcal{L})$, where $\nu_1, \nu_2 \in I_\sigma(\mathcal{L})$, it follows that $\nu_1 = \nu_2$.

Proof. (a) Let $\mu_1, \mu_2 \in I(\mathcal{L})$ such that $\mu_1 \leq \mu_2(\mathcal{L})$; there exist $\underline{\mu}_1, \underline{\mu}_2 \in I(\mathcal{W}_\sigma(\mathcal{L}))$ and $\underline{\mu}_1(W_\sigma(L)) = \mu_1(L)$, $\underline{\mu}_2(W_\sigma(L)) = \mu_2(L)$ for all $L \in \mathcal{L}$. Moreover, $\mu_1(L) \leq \mu_2(L) \Rightarrow \underline{\mu}_1 \leq \underline{\mu}_2$ on $\mathcal{W}_\sigma(\mathcal{L})$. Suppose $\mathcal{W}_\sigma(\mathcal{L})$ is regular; then $S(\underline{\mu}_1) = S(\underline{\mu}_2)$, where

$$S(\underline{\mu}_1) = \bigcap \{W_\sigma(L) \in \mathcal{W}_\sigma(\mathcal{L}) \mid \underline{\mu}_1(W_\sigma(L)) = 1, L \in \mathcal{L}\}.$$

Let now $\nu \in I_R^\sigma(\mathcal{L})$ with $\mu_1 \leq \nu(\mathcal{L})$. We have $\underline{\nu} \in I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$ and $\underline{\mu}_1 \leq \underline{\nu}(\mathcal{W}_\sigma(\mathcal{L}))$, therefore $S(\underline{\nu}) \subset S(\underline{\mu}_1) = S(\underline{\mu}_2)$; hence $\underline{\mu}_2 \leq \underline{\nu}(\mathcal{W}_\sigma(\mathcal{L}))$ i.e. $\mu_2 \leq \nu(\mathcal{L})$.

Conversely, let $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\nu \in I_R^\sigma(\mathcal{L})$ such that if $\mu_1 \leq \mu_2(\mathcal{L})$ and $\mu_1 \leq \nu(\mathcal{L})$ then $\mu_2 \leq \nu(\mathcal{L})$. Let now $\lambda_1, \lambda_2 \in I(\mathcal{W}_\sigma(\mathcal{L}))$ and assume $\lambda_1 \leq \lambda_2(\mathcal{W}_\sigma(\mathcal{L}))$. Then $\lambda_1 = \underline{\mu}_1$ and $\lambda_2 = \underline{\mu}_2$ where $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\underline{\mu}_1 \leq \underline{\mu}_2(\mathcal{W}_\sigma(\mathcal{L}))$ i.e. $\mu_1 \leq \mu_2(\mathcal{L})$. Now $S(\underline{\mu}_2) \subset S(\underline{\mu}_1)$; if $\lambda \in S(\underline{\mu}_1)$, then clearly $\lambda \in I_R^\sigma(\mathcal{L})$ and $\mu_1 \leq \lambda(\mathcal{L})$. Hence, by the assumption, $\mu_2 \leq \lambda(\mathcal{L})$ which implies $\lambda \in S(\underline{\mu}_2)$.

(b) Suppose $[I_R^\sigma(\mathcal{L}), t\mathcal{W}_\sigma(\mathcal{L})]$ is T_2 (which implies that $\mathcal{W}_\sigma(\mathcal{L})$ is T_2), and let μ, ν_1, ν_2 as above. Then $\underline{\mu} \leq \underline{\nu}_1(\mathcal{W}_\sigma(\mathcal{L}))$ where $\underline{\mu} \in I(\mathcal{W}_\sigma(\mathcal{L}))$ and $\underline{\nu}_1 \in I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$, which implies $\nu_1 \in S(\underline{\nu}_1) \subset S(\underline{\mu})$. Also $\underline{\mu} \leq \underline{\nu}_2(\mathcal{W}_\sigma(\mathcal{L}))$ where $\underline{\nu}_2 \in I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$, which implies $\nu_2 \in S(\underline{\nu}_2) \subset S(\underline{\mu})$. Recall that $\mathcal{L}$ is T_2 if, and only if, for each $\mu \in I(\mathcal{L})$, $S(\underline{\mu}) = \emptyset$ or a singleton, hence since $\mathcal{W}_\sigma(\mathcal{L})$ is T_2 it follows $\nu_1 = \nu_2$.

Conversely, assume that for $\mu \in I(\mathcal{L})$ and $\nu_1, \nu_2 \in I_R^\sigma(\mathcal{L})$, if $\mu \leq \nu_1, \nu_2(\mathcal{L})$, it follows that $\nu_1 = \nu_2$. Suppose $S(\underline{\mu}) \neq \emptyset$, where $\mu \in I(\mathcal{L})$, $\lambda \in I(\mathcal{W}_\sigma(\mathcal{L}))$ and $\lambda = \underline{\mu}$. If $\nu_1, \nu_2 \in S(\underline{\mu})$ then $\mu \leq \nu_1, \nu_2(\mathcal{L})$ i.e. $\nu_1 = \nu_2$. Therefore $\mathcal{W}_\sigma(\mathcal{L})$ is T_2 and thus $t\mathcal{W}_\sigma(\mathcal{L})$ is T_2 .

Clearly, the proofs for (c) and (d) are similar and we omit them. $\square$

Theorem 3.2. Consider $I_\sigma(\mathcal{L})$ and $\mathcal{V}_\sigma(\mathcal{L})$. $\mathcal{V}_\sigma(\mathcal{L})$ is regular if, and only if, $I_\sigma(\mathcal{L}) = I_R^\sigma(\mathcal{L})$. (We note here that the assumption of regularity for $\mathcal{V}_\sigma(\mathcal{L})$ in $I_\sigma(\mathcal{L})$ is very strong, since implies that any σ -smooth measure is also $\mathcal{L}$ -regular).

Proof. Suppose $I_\sigma(\mathcal{L}) = I_R^\sigma(\mathcal{L})$. Then $\mathcal{V}_\sigma(\mathcal{L}) = \mathcal{W}_\sigma(\mathcal{L})$. Now let $\mu_1, \mu_2 \in I(\mathcal{L})$, $\nu \in I_\sigma(\mathcal{L})$ and $\mu_1 \leq \mu_2(\mathcal{L})$, $\mu_1 \leq \nu(\mathcal{L})$. Then, since $I_\sigma(\mathcal{L}) = I_R^\sigma(\mathcal{L})$, $\mu_1 \in I_R^\sigma(\mathcal{L})$ so $\mu_1 = \mu_2$ and $\mu_1 = \nu$.

Conversely, suppose $\mathcal{V}_\sigma(\mathcal{L})$ is regular and let $\mu \in I_\sigma(\mathcal{L})$; there exists $\nu \in I_R(\mathcal{L})$ such that $\underline{\mu} \leq \underline{\nu}(\mathcal{V}_\sigma(\mathcal{L}))$, where $\underline{\mu}, \underline{\nu} \in I_\sigma(\mathcal{V}_\sigma(\mathcal{L}))$. But $S(\underline{\mu}) = S(\underline{\nu})$ since $\mathcal{V}_\sigma(\mathcal{L})$ is regular. Hence $\mu \in S(\underline{\nu})$ i.e. $\nu \leq \mu(\mathcal{L})$. It follows $\mu = \nu$ and then $\mu \in I_R^\sigma(\mathcal{L})$. $\square$

4. Normal, slightly normal, mildly normal and Lindelöf lattices. In this section we wish to consider normality and related questions as well as Lindelöf properties concerning the lattices $\mathcal{W}_\sigma(\mathcal{L})$ and $\mathcal{V}_\sigma(\mathcal{L})$. Related details on normality can be found in [6, 10, 13, 14]. First, we recall some definitions.

Definition 7. (a) $\mathcal{L}$ is *slightly normal* if for all $\mu \in I_\sigma(\mathcal{L}')$, there exists a unique $\nu \in I_R(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$.

(b) $\mathcal{L}$ is *mildly normal* if for all $\mu \in I_\sigma(\mathcal{L})$, there exists a unique $\nu \in I_R(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$.

(c) $\mathcal{L}$ is *almost countably compact* if $\mu \in I_R(\mathcal{L}')$ implies $\mu \in I_\sigma(\mathcal{L})$.

Theorem 4.1. Suppose $\mathcal{L}$ is disjunctive. Then we have the following:

(a) Consider $I_R^\sigma(\mathcal{L})$ and $\mathcal{W}_\sigma(\mathcal{L})$ and suppose $\mathcal{L}$ is Lindelöf and satisfies the condition:

for all $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\nu \in I_R^\sigma(\mathcal{L})$, if $\mu_1 \leq \mu_2, \nu(\mathcal{L})$ then $\mu_2 \leq \nu(\mathcal{L})$.

Then $\mathcal{W}_\sigma(\mathcal{L})$ is slightly and mildly normal.

(b) If $\mathcal{L}$ is complement generated then $\mathcal{W}_\sigma(\mathcal{L})$ is slightly normal.

(c) If $\mathcal{L}$ is almost countably compact and mildly normal then $\mathcal{W}_\sigma(\mathcal{L})$ is normal.

Proof. (a) $\mathcal{L}$ disjunctive and Lindelöf implies $\mathcal{W}_\sigma(\mathcal{L})$ Lindelöf. Also, by Theorem 3.1, it follows that $\mathcal{W}_\sigma(\mathcal{L})$ is regular. Then $\mathcal{W}_\sigma(\mathcal{L})$ is slightly and mildly normal (see[6]).

(b) The assumption that $\mathcal{L}$ is complement generated implies

$$L = \bigcap_{n=1}^{\infty} L'_n, \quad L \text{ and } L_n \in \mathcal{L} \text{ for all } n.$$

Then:

$$W_\sigma(L) = W_\sigma\left(\bigcap_{n=1}^{\infty} L'_n\right) = \bigcap_{n=1}^{\infty} W_\sigma(L'_n) = \bigcap_{n=1}^{\infty} [W_\sigma(L_n)]'.$$

Hence $\mathcal{W}_\sigma(\mathcal{L})$ complement generated which implies $\mathcal{W}_\sigma(\mathcal{L})$ slightly normal (see [6]).

(c) By the assumption, for any $\mu \in I_R(\mathcal{L}')$ it follows $\mu \in I_\sigma(\mathcal{L})$ and then there exists a unique $\nu \in I_R(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$. Let $\mu \in I(\mathcal{L})$ such that $\mu \leq \lambda(\mathcal{L}')$ with $\lambda \in I_R(\mathcal{L}')$. Also $\lambda \in I_\sigma(\mathcal{L})$ and $\lambda \leq \mu \leq \nu_1(\mathcal{L})$ with $\nu_1 \in I_R(\mathcal{L})$, unique. Therefore if $\mu \leq \nu_2(\mathcal{L})$ with $\nu_2 \in I_R(\mathcal{L})$, then $\lambda \leq \mu \leq \nu_2(\mathcal{L})$, and so $\nu_1 = \nu_2$. Hence $\mathcal{L}$ is normal and also $\mathcal{W}_\sigma(\mathcal{L})$ is normal. $\square$

Remark. Consider $I_\sigma(\mathcal{L})$ and $\mathcal{V}_\sigma(\mathcal{L})$ with $\mathcal{L}$ Lindelöf. If for all $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\nu \in I_\sigma(\mathcal{L})$ such that if $\mu_1 \leq \mu_2(\mathcal{L})$ and $\mu_1 \leq \nu(\mathcal{L})$ it follows that $\mu_2 \leq \nu(\mathcal{L})$, then $\mathcal{V}_\sigma(\mathcal{L})$ is slightly and mildly normal.

We next consider the following conditions to be satisfied by a lattice $\mathcal{L}$ and obtain results pertaining to them:

Condition (1): For any $\pi \in \mathcal{P}_\sigma(\mathcal{L})$, there exists $\nu \in I_\sigma(\mathcal{L})$ such that $\pi \leq \nu(\mathcal{L})$.

Condition (2): For any $\pi \in \mathcal{P}_\sigma(\mathcal{L})$, there exists $\nu \in I_R^\sigma(\mathcal{L})$ such that $\pi \leq \nu(\mathcal{L})$.

Theorem 4.2. (a) If condition (1) is satisfied and $\mathcal{L}$ is prime complete then $\mathcal{L}$ is Lindelöf.

(b) If $\mathcal{L}$ is Lindelöf then condition (1) holds.

(c) $\mathcal{L}$ satisfies condition (1) if, and only if, $[I_\sigma(\mathcal{L}), t\mathcal{V}_\sigma(\mathcal{L})]$ is Lindelöf.

Proof. (a) Let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$ be a σ -smooth premeasure on $\mathcal{L}$. By condition (1), there exists $\nu \in I_\sigma(\mathcal{L})$ and $\pi \leq \nu(\mathcal{L})$. $\mathcal{L}$ prime complete implies $S(\nu) \neq \emptyset$ and then $S(\pi) \neq \emptyset$.

(b) Let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$; since $\mathcal{L}$ is Lindelöf, $S(\pi) \neq \emptyset$, therefore there exists $x \in X$ such that $x \in S(\pi)$. Then $\pi \leq \mu_x(\mathcal{L})$ and $\mu_x \in I_\sigma(\mathcal{L})$.

(c) Suppose that $\mathcal{L}$ satisfies condition (1). Let $\underline{\pi} \in \mathcal{P}_\sigma(\mathcal{V}_\sigma(\mathcal{L}))$ and define $\pi(L) = \underline{\pi}(V_\sigma(L))$, $L \in \mathcal{L}$. If $L_n \downarrow \emptyset$, $L_n \in \mathcal{L}$ then $V_\sigma(L_n) \downarrow \emptyset$ and $\pi(L_n) = \underline{\pi}(V_\sigma(L_n)) \rightarrow 0$, i.e. $\pi \in \mathcal{P}_\sigma(\mathcal{L})$. By condition (1), there exists $\nu \in I_\sigma(\mathcal{L})$ such that $\pi \leq \nu(\mathcal{L})$. Hence $\underline{\nu} \in I_\sigma(\mathcal{V}_\sigma(\mathcal{L}))$ and $\underline{\pi} \leq \underline{\nu}(\mathcal{V}_\sigma(\mathcal{L}))$. Therefore $\mathcal{V}_\sigma(\mathcal{L})$ satisfies condition (1). Next, we show that $\mathcal{V}_\sigma(\mathcal{L})$ is prime complete; this will imply $\mathcal{V}_\sigma(\mathcal{L})$ Lindelöf and then $t\mathcal{V}_\sigma(\mathcal{L})$ Lindelöf. Let

$$S(\underline{\nu}) = \bigcap_{a \in A} \{V_\sigma(L_a) \mid \underline{\nu}(V_\sigma(L_a)) = 1, L_a \in \mathcal{L}\}$$

where A is any index set. But $\underline{\nu}(V_\sigma(L)) = 1$ if, and only if, $\nu(L) = 1$ if, and only if, $\nu \in V_\sigma(L) = \{\mu \in I_\sigma(\mathcal{L}) \mid \mu(L) = 1, L \in \mathcal{L}\}$. Hence $S(\underline{\nu}) \neq \emptyset$.

Conversely, let $[I_\sigma(\mathcal{L}), t\mathcal{V}_\sigma(\mathcal{L})]$ be Lindelöf, let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$ and define $\underline{\pi}(V_\sigma(L)) = \pi(L)$, $L \in \mathcal{L}$. Then $V_\sigma(L_n) \downarrow \emptyset$ implies $L_n \downarrow \emptyset$ and $\pi(L_n) = \underline{\pi}(V_\sigma(L_n)) \rightarrow 0$, hence $\underline{\pi} \in \mathcal{P}_\sigma(\mathcal{V}_\sigma(\mathcal{L}))$. But $t\mathcal{V}_\sigma(\mathcal{L})$ Lindelöf implies $\mathcal{V}_\sigma(\mathcal{L})$ Lindelöf and then $\mathcal{V}_\sigma(\mathcal{L})$ satisfies condition (1); hence there exists

$$\underline{\nu} \in I_\sigma(\mathcal{V}_\sigma(\mathcal{L})) \text{ such that } \underline{\pi} \leq \underline{\nu}(\mathcal{V}_\sigma(\mathcal{L})), \text{ where } \underline{\nu}(V_\sigma(L)) = \nu(L), L \in \mathcal{L}.$$

Now $\pi(L) = 1$ implies $\underline{\pi}(V_\sigma(L)) = 1$ i.e. $\nu(L) = 1$, $L \in \mathcal{L}$, so $\pi \leq \nu(\mathcal{L})$. $\square$

Theorem 4.3. (a) If $\mathcal{L}$ satisfies condition (2) and is replete then $\mathcal{L}$ is Lindelöf.

(b) If $\mathcal{L}$ is a countably compact lattice then $\mathcal{L}$ satisfies condition (2).

(c) If $\mathcal{L}$ is disjunctive and Lindelöf then $\mathcal{L}$ satisfies condition (2).

(d) If $\mathcal{L}$ is disjunctive then $\mathcal{L}$ satisfies condition (2) if, and only if, $[I_R^\sigma(\mathcal{L}), t\mathcal{W}_\sigma(\mathcal{L})]$ is Lindelöf.

Proof. (a) Let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$ and $\nu \in I_R^\sigma(\mathcal{L})$ such that $\pi \leq \nu(\mathcal{L})$. $\mathcal{L}$ replete implies $S(\nu) = \bigcap_{a \in A} \{L_a \in \mathcal{L} \mid \nu(L_a) = 1\} \neq \emptyset$ (where A is just a set of indexes), and then $S(\nu) \subset S(\pi) \neq \emptyset$. But π has the countable intersection property therefore

$$\bigcap_{a=1}^{\infty} \{L \in \mathcal{L} \mid \pi(L_a) = 1\} \neq \emptyset \text{ i.e. } \mathcal{L} \text{ is Lindelöf.}$$

(b) Let $\{L_a\}_{a=1, \dots, \infty}$ be a collection of subsets of X such that $\bigcap_{a=1}^{\infty} \{L_a\} \neq \emptyset$. Since $\mathcal{L}$ is countably compact, $\bigcap_{a=1}^{\infty} \{L_a\} \neq \emptyset$ and then $\{L_a\}_a$ is a filter base which generates an $\mathcal{L}$ -filter with the countable intersection property, $\pi \in \mathcal{P}_\sigma(\mathcal{L})$. We enlarge it to F , an $\mathcal{L}$ -ultrafilter with the countable intersection property. To F it corresponds uniquely $\nu \in I_R^\sigma(\mathcal{L})$. Now from a filter-ultrafilter argument it follows that $\pi \leq \nu(\mathcal{L})$.

(c) Let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$ and $\{L_a\}_{a \in A}$, a family of subsets of X . Then $\bigcap_{a=1}^{\infty} \{L_a \mid \pi(L_a) = 1\} \neq \emptyset$ and since $\mathcal{L}$ is Lindelöf, $\bigcap_{a \in A} \{L_a \in \mathcal{L} \mid \pi(L_a) = 1\} = S(\pi) \neq \emptyset$. Let $x \in S(\pi)$ and consider μ_x . Clearly, $\pi \leq \mu_x(\mathcal{L})$. From the countable intersection property of π it follows that $\mu_x \in I_\sigma(\mathcal{L})$ and since $\mathcal{L}$ is disjunctive $\mu_x \in I_R(\mathcal{L})$.

(d) Let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$; there exists $\mu \in I_R^\sigma(\mathcal{L})$ and $\pi \leq \mu(\mathcal{L})$. Then $\underline{\pi} \leq \underline{\mu}(\mathcal{W}_\sigma(\mathcal{L}))$ with $\underline{\pi} \in \mathcal{P}_\sigma(\mathcal{W}_\sigma(\mathcal{L}))$ and $\underline{\mu} \in I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$, therefore $\mathcal{W}_\sigma(\mathcal{L})$ satisfies condition (2). Since $\mathcal{L}$ is disjunctive $\mathcal{W}_\sigma(\mathcal{L})$ is replete. Then by (a) it follows that $\mathcal{W}_\sigma(\mathcal{L})$ is Lindelöf and then $t\mathcal{W}_\sigma(\mathcal{L})$ is Lindelöf.

Conversely, $t\mathcal{W}_\sigma(\mathcal{L})$ Lindelöf implies that $\mathcal{W}_\sigma(\mathcal{L})$ is Lindelöf and since $\mathcal{W}_\sigma(\mathcal{L})$ is also disjunctive, by (c) it follows that $\mathcal{W}_\sigma(\mathcal{L})$ satisfies condition (2). Therefore for $\underline{\pi} \in \mathcal{P}_\sigma(\mathcal{W}_\sigma(\mathcal{L}))$ there exists $\underline{\mu} \in I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$ such that $\underline{\pi} \leq \underline{\mu}(\mathcal{W}_\sigma(\mathcal{L}))$. To $\underline{\pi}$ and $\underline{\mu}$ correspond $\pi \in \mathcal{P}_\sigma(\mathcal{L})$ and $\mu \in I_R^\sigma(\mathcal{L})$ such that $\pi \leq \mu(\mathcal{L})$. $\square$

5. Prime complete and countably compact lattices. In this section, we investigate the equivalence and consequences of stronger lattice completeness assumption. We start by defining other conditions for the lattice $\mathcal{L}$.

Condition (3): for any $\mu \in I_\sigma(\mathcal{L})$ there exists $\nu \in I_R^\sigma(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$.

Condition (4): for any $\mu \in I_\sigma(\mathcal{L}')$ there exists $\nu \in I_R^\sigma(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$.

Theorem 5.1. *Let $\mathcal{L}$ be a disjunctive lattice. Then $\mathcal{L}$ satisfies condition (3) if, and only if, $\mathcal{W}_\sigma(\mathcal{L})$ is prime complete.*

Proof. Let $\mu \in I_\sigma(\mathcal{L})$ and the associated induced measure $\underline{\mu} \in I_\sigma(\mathcal{W}_\sigma(\mathcal{L}))$ with $\underline{\mu}(\mathcal{W}_\sigma(L)) = \mu(L)$, $L \in \mathcal{L}$. If $\mathcal{W}_\sigma(\mathcal{L})$ is prime complete, $S(\underline{\mu}) \neq \emptyset$ and then there exists $\nu \in S(\underline{\mu})$, $\nu \in I_R^\sigma(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$.

Conversely, let $\underline{\mu} \in I_\sigma(\mathcal{W}_\sigma(\mathcal{L}))$ and consider the associated $\mu \in I_\sigma(\mathcal{L})$ such that $\underline{\mu}(\mathcal{W}_\sigma(L)) = \mu(L)$. For $\mu \in I_\sigma(\mathcal{L})$, there exists $\nu \in I_R^\sigma(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$. Therefore $\underline{\nu} \in I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$ and $\underline{\mu} \leq \underline{\nu}(\mathcal{W}_\sigma(\mathcal{L}))$ which implies $S(\underline{\nu}) \subset S(\underline{\mu})$ and since $\mathcal{W}_\sigma(\mathcal{L})$ is replete, $S(\underline{\nu}) \neq \emptyset$. $\square$

Theorem 5.2. (a) *Let $\mathcal{L}$ be disjunctive, almost countably compact and mildly normal and let $\mathcal{W}_\sigma(\mathcal{L})$ be prime complete. Then $\mathcal{L}$ is countably compact.*

(b) *Let $\mathcal{L}$ be disjunctive, regular, Lindelöf, almost countably compact and let $\mathcal{W}_\sigma(\mathcal{L})$ be prime complete. Then $\mathcal{L}$ is countably compact.*

Proof. (a) We must show that $I_R(\mathcal{L}) = I_R^\sigma(\mathcal{L})$. Let $\mu \in I_R(\mathcal{L})$; we have $\mu \leq \nu(\mathcal{L}')$ where $\nu \in I_R(\mathcal{L}')$. Since $\mathcal{L}$ is almost countably compact we have $\nu \leq \mu(\mathcal{L})$ with $\nu \in I_\sigma(\mathcal{L})$. But $\mathcal{W}_\sigma(\mathcal{L})$ is prime complete and by Theorem 5.1, there exists $\lambda \in I_R^\sigma(\mathcal{L})$ such that $\nu \leq \lambda(\mathcal{L})$. $\mathcal{L}$ almost countably compact and mildly normal implies $\mathcal{L}$ normal (see [6]). By the normality of $\mathcal{L}$, the $\mathcal{L}$ -regular measure μ such that $\nu \leq \mu$ must be unique, hence $\mu = \lambda \in I_R^\sigma(\mathcal{L})$.

(b) $\mathcal{L}$ regular and Lindelöf implies $\mathcal{L}$ mildly normal and by the above it follows that $\mathcal{L}$ is countably compact. $\square$

Theorem 5.3. *Suppose that $[I_\sigma(\mathcal{L}), t\mathcal{V}_\sigma(\mathcal{L})]$ is T_1 and that $\mathcal{L}$ is disjunctive and $\mathcal{W}_\sigma(\mathcal{L})$ prime complete. Then $I_\sigma(\mathcal{L}) = I_R^\sigma(\mathcal{L})$.*

Proof. Since $[I_\sigma(\mathcal{L}), t\mathcal{V}_\sigma(\mathcal{L})]$ is T_1 , given $\lambda \neq \mu$ with $\lambda, \mu \in I_\sigma(\mathcal{L})$, there exist $L_1, L_2 \in \mathcal{L}$ such that $\lambda \in V_\sigma(L_1')$, $\mu \notin V_\sigma(L_1')$ and $\mu \in V_\sigma(L_2')$, $\lambda \notin V_\sigma(L_2')$. Therefore $\lambda(L_1') = 1$, $\mu(L_1') = 0$ or $\lambda(L_1) = 0$, $\mu(L_1) = 1$ and $\mu(L_2') = 1$, $\lambda(L_2') = 0$ or $\mu(L_2) = 0$, $\lambda(L_2) = 1$. Since $\mathcal{W}_\sigma(\mathcal{L})$ prime complete, given $\mu \in I_\sigma(\mathcal{L})$ there exists $\nu \in I_R^\sigma(\mathcal{L})$ with $\mu \leq \nu(\mathcal{L})$. If $\mu \neq \nu$, by above exists $L \in \mathcal{L}$ such that $\nu(L) = 0$ and $\mu(L) = 1$. Contradiction, hence $\mu = \nu$ and $I_\sigma(\mathcal{L}) = I_R^\sigma(\mathcal{L})$. $\square$

Remarks. (a) The converse of Theorem 5.3 is true i.e. if $I_\sigma(\mathcal{L}) = I_R^\sigma(\mathcal{L})$ then $\mathcal{V}_\sigma(\mathcal{L}) = \mathcal{W}_\sigma(\mathcal{L})$ is T_1 and since $\mathcal{V}_\sigma(\mathcal{L})$ is prime complete, $\mathcal{W}_\sigma(\mathcal{L})$ is prime complete, too.

(b) If $\mathcal{L}$ is regular and Lindelöf then $\mathcal{L}$ satisfies condition (3).

- (c) If $\mathcal{L}$ satisfies condition (3) and is replete then $\mathcal{L}$ is prime complete.
 - (d) If $\mathcal{L}$ satisfies condition (4) and is replete then $\mathcal{L}'$ is prime complete.
- (For the proofs see [14]).

Theorem 5.4. *Let $\mathcal{L}$ be a disjunctive lattice of subsets of X .*

- (a) *If $\mathcal{L}$ satisfies condition (4) then $\mathcal{W}_\sigma(\mathcal{L})'$ is prime complete.*
- (b) *If $\mathcal{W}_\sigma(\mathcal{L})'$ is prime complete then $\mathcal{L}'$ satisfies condition (3).*

Proof. (a) Let $\underline{\mu} \in I_\sigma(\mathcal{W}_\sigma(\mathcal{L})')$ and the corresponding $\mu \in I_\sigma(\mathcal{L}')$. There exists $\nu \in I_R^\sigma(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$. Then $\underline{\mu} \leq \underline{\nu}(\mathcal{W}_\sigma(\mathcal{L}))$ and since $\mathcal{W}_\sigma(\mathcal{L})$ is replete, $S(\underline{\nu}) \neq \emptyset$, which implies $S(\underline{\mu}) \neq \emptyset$.

(b) Let $\mu \in I_\sigma(\mathcal{L}')$ and consider the associated $\underline{\mu} \in I_\sigma(\mathcal{W}_\sigma(\mathcal{L})')$. $S(\underline{\mu}) \neq \emptyset$, since $\mathcal{W}_\sigma(\mathcal{L})'$ is prime complete. Therefore there exists $\nu \in S(\underline{\mu})$, $\nu \in I_R^\sigma(\mathcal{L}')$ and $\mu \leq \nu(\mathcal{L}')$. $\square$

6. Strongly σ -smooth measures. Here we consider another general Wallman space, consisting of strongly σ -smooth measures and analyze the relevant lattice in detail. First, we introduce the notion of strongly σ -smooth measure:

Definition 8: strongly σ -smoothness and weakly prime completeness. A measure $\mu \in I(\mathcal{L})$ is strongly σ -smooth on $\mathcal{L}$ if, and only if, for any non increasing sequence $\{L_n\}$, $L_n \in \mathcal{L}$, if $\bigcap L_n \in \mathcal{L}$ then $\mu(\bigcap L_n) = \inf_n \mu(L_n) = \lim_n \mu(L_n)$.

We denote by $I(\sigma, \mathcal{L})$ the set of strongly σ -smooth nontrivial zero-one valued measures on $\mathcal{L}$.

We call the lattice $\mathcal{L}$ *weakly prime complete* if for $\mu \in I(\sigma, \mathcal{L})$, $S(\mu) \neq \emptyset$.

We can now define another Wallman topological space: $[I(\sigma, \mathcal{L}), t\mathcal{V}(\sigma, \mathcal{L})]$ where

$$\mathcal{V}(\sigma, \mathcal{L}) = \{V(\sigma, L) \mid L \in \mathcal{L}\} \text{ with } V(\sigma, L) = \{\mu \in I(\sigma, \mathcal{L}) \mid \mu(L) = 1, L \in \mathcal{L}\}.$$

We recall a few statements on σ -smoothness that will be used throughout this section for the reader's convenience (see [10]).

- (a) $I^\sigma(\mathcal{L}) \subset I(\sigma, \mathcal{L}) \subset I_\sigma(\mathcal{L})$;
- (b) $\mathcal{L}$ normal and complement generated implies $I(\sigma, \mathcal{L}) \subset I_R^\sigma(\mathcal{L})$;
- (c) $I_R^\sigma(\mathcal{L}) \subset I(\sigma, \mathcal{L})$.

Now we define a condition similar to condition (1) introduced in Section 4 and proceed to prove results, some of them analogous to those proved in Section 4.

Condition (5): For any $\pi \in \mathcal{P}_\sigma(\mathcal{L})$, there exists $\nu \in I(\sigma, \mathcal{L})$ such that $\pi \leq \nu(\mathcal{L})$.

Theorem 6.1. (a) *If condition (5) holds and if $\mathcal{L}$ is weakly prime complete, $\mathcal{L}$ is Lindelöf.*
 (b) *If $\mathcal{L}$ is Lindelöf then condition (5) holds.*

Proof. (a) Let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$. By condition (5) there exists $\nu \in I(\sigma, \mathcal{L})$ such that $\pi \leq \nu(\mathcal{L})$. $\mathcal{L}$ weakly prime complete implies that $S(\nu) \neq \emptyset$. $\pi \leq \nu$ implies $S(\pi) \supset S(\nu) \neq \emptyset$. Hence $S(\pi) \neq \emptyset$, which in turn, implies that $\mathcal{L}$ is Lindelöf.

(b) Let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$. Since $\mathcal{L}$ is Lindelöf, $S(\pi) \neq \emptyset$. Let $x \in S(\pi)$; then $\pi \leq \mu_x(\mathcal{L})$ and $\mu_x \in I(\sigma, \mathcal{L})$, and condition (5) holds. $\square$

Theorem 6.2. $\mathcal{L}$ satisfies condition (5) if, and only if, $[I(\sigma, \mathcal{L}), t\mathcal{V}(\sigma, \mathcal{L})]$ is Lindelöf.

Proof. Suppose $\mathcal{L}$ satisfies condition (5); we want to show first that $\mathcal{V}(\sigma, \mathcal{L})$ satisfies condition (5) and then that is weakly prime complete. Let $\underline{\pi} \in \mathcal{P}_\sigma(\mathcal{V}(\sigma, \mathcal{L}))$ and define $\pi(L) = \underline{\pi}(V(\sigma, L))$, $L \in \mathcal{L}$. If $L_n \downarrow \emptyset$, $L_n \in \mathcal{L}$, then $V(\sigma, L_n) \downarrow \emptyset$ and $\lim_n \pi(L_n) = \lim_n \underline{\pi}(V(\sigma, L_n)) = 0$, hence $\pi \in \mathcal{P}_\sigma(\mathcal{L})$. By condition (5) there exists $\nu \in I(\sigma, \mathcal{L})$ such that $\pi \leq \nu(\mathcal{L})$. Hence $\underline{\nu} \in I(\sigma, \mathcal{V}(\sigma, \mathcal{L}))$ and $\underline{\pi} \leq \underline{\nu}(\mathcal{V}(\sigma, \mathcal{L}))$, where $\underline{\nu}(V(\sigma, L)) = \nu(L)$, $L \in \mathcal{L}$. Next, let

$$S(\underline{\nu}) = \bigcap_{a \in A} \{V(\sigma, L_a) \mid \underline{\nu}(V(\sigma, L_a)) = 1, L_a \in \mathcal{L}\}.$$

But $\underline{\nu}(V(\sigma, L)) = 1$ if, and only if, $\nu(L) = 1$ if, and only if, $\nu \in V(\sigma, L)$. Hence $V(\sigma, L) \neq \emptyset$ which implies $S(\underline{\nu}) \neq \emptyset$, therefore $\mathcal{V}(\sigma, \mathcal{L})$ is Lindelöf, and then $t\mathcal{V}(\sigma, \mathcal{L})$ is Lindelöf.

Conversely, assume $[I(\sigma, \mathcal{L}), t\mathcal{V}(\sigma, \mathcal{L})]$ is Lindelöf and let $\pi \in \mathcal{P}_\sigma(\mathcal{L})$. Define $\underline{\pi}(V(\sigma, L)) = \pi(L)$, $L \in \mathcal{L}$. Then $V(\sigma, L_n) \downarrow \emptyset$ which implies $L_n \downarrow \emptyset$ and $\lim_n \pi(L_n) = \lim_n \underline{\pi}(V(\sigma, L_n)) = 0$ i.e. $\underline{\pi} \in \mathcal{P}_\sigma(\mathcal{V}(\sigma, \mathcal{L}))$. $t\mathcal{V}(\sigma, \mathcal{L})$ Lindelöf implies $\mathcal{V}(\sigma, \mathcal{L})$ Lindelöf and then $\mathcal{V}(\sigma, \mathcal{L})$ satisfies condition (5). Hence, there exists $\underline{\nu} \in I(\sigma, \mathcal{V}(\sigma, \mathcal{L}))$ and $\underline{\pi} \leq \underline{\nu}(\mathcal{V}(\sigma, \mathcal{L}))$, where $\underline{\nu}(V(\sigma, L)) = \nu(L)$, $L \in \mathcal{L}$. Therefore $\pi \leq \nu(\mathcal{L})$. $\square$

Theorem 6.3. Consider $I(\sigma, \mathcal{L})$ and $\mathcal{V}(\sigma, \mathcal{L})$. $\mathcal{V}(\sigma, \mathcal{L})$ is regular if, and only if, for all $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\nu \in I(\sigma, \mathcal{L})$, if $\mu_1 \leq \mu_2, \nu(\mathcal{L})$ then $\mu_2 \leq \nu(\mathcal{L})$.

Proof. For $\mu_1, \mu_2 \in I(\mathcal{L})$ we have $\underline{\mu}_1, \underline{\mu}_2 \in I(\mathcal{W}_\sigma(\mathcal{L}))$ and then $\underline{\mu}_1, \underline{\mu}_2 \in I(\mathcal{V}(\sigma, \mathcal{L}))$. If $\mathcal{V}(\sigma, \mathcal{L})$ is regular then $S(\underline{\mu}_1) = S(\underline{\mu}_2)$, where

$$S(\underline{\mu}_1) = \bigcap \{V(\sigma, L) \in \mathcal{V}(\sigma, \mathcal{L}) \mid \underline{\mu}_1(V(\sigma, L)) = 1, L \in \mathcal{L}\}.$$

Let $\nu \in I(\sigma, \mathcal{L})$; $\underline{\nu} \in I(\sigma, \mathcal{V}(\sigma, \mathcal{L}))$ and $\underline{\mu}_1 \leq \underline{\nu}(\mathcal{V}(\sigma, \mathcal{L}))$. Then $\nu \in S(\underline{\mu}_2)$ i.e. $\mu_2 \leq \nu(\mathcal{L})$.

Conversely, suppose $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\nu \in I(\sigma, \mathcal{L})$, such that if $\mu_1 \leq \mu_2, \nu(\mathcal{L})$ then $\mu_2 \leq \nu(\mathcal{L})$. Let $\lambda_1, \lambda_2 \in I(\mathcal{V}(\sigma, \mathcal{L}))$ and $\lambda_1 \leq \lambda_2(\mathcal{V}(\sigma, \mathcal{L}))$. Then $\lambda_1 = \underline{\mu}_1$ and $\lambda_2 = \underline{\mu}_2$ with $\mu_1, \mu_2 \in I(\mathcal{L})$. Thus $\underline{\mu}_1 \leq \underline{\mu}_2 \in (\mathcal{V}(\sigma, \mathcal{L}))$ which implies $\mu_1 \leq \mu_2(\mathcal{L})$, hence $S(\underline{\mu}_2) \subset S(\underline{\mu}_1)$. If $\lambda \in S(\underline{\mu}_1)$ then clearly $\lambda \in I(\sigma, \mathcal{L})$ and $\mu_1 \leq \lambda(\mathcal{L})$. By assumption, $\mu_2 \leq \lambda(\mathcal{L})$ and then $\lambda \in S(\underline{\mu}_2)$. Hence $S(\underline{\mu}_2) = S(\underline{\mu}_1)$ and $\mathcal{V}(\sigma, \mathcal{L})$ is regular. $\square$

Theorem 6.4. Consider $I(\sigma, \mathcal{L})$ and $\mathcal{V}(\sigma, \mathcal{L})$. If $\mathcal{V}(\sigma, \mathcal{L})$ is regular, then $I(\sigma, \mathcal{L}) = I_R^\sigma(\mathcal{L})$.

Proof. Let $\mu \in I(\sigma, \mathcal{L})$; there exists $\nu \in I_R(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$, hence $\underline{\mu} \leq \underline{\nu}(\mathcal{V}(\sigma, \mathcal{L}))$ where $\underline{\mu} \in I(\sigma, \mathcal{V}(\sigma, \mathcal{L}))$ and $\underline{\nu} \in I_R(\mathcal{V}(\sigma, \mathcal{L}))$. $\mathcal{V}(\sigma, \mathcal{L})$ is regular, therefore $S(\underline{\mu}) = S(\underline{\nu})$, hence $\nu \leq \mu(\mathcal{L})$. It follows $\mu = \nu(\mathcal{L})$ and since $\nu \in I_R(\mathcal{L})$, $I(\sigma, \mathcal{L}) \subset I_\sigma(\mathcal{L})$. Therefore $\mu \in I_R(\mathcal{L})$, $I_\sigma(\mathcal{L})$ and then $\mu \in I_R^\sigma(\mathcal{L})$. $\square$

Theorem 6.5. Consider $I(\sigma, \mathcal{L})$ and $\mathcal{V}(\sigma, \mathcal{L})$ with $\mathcal{L}$ Lindelöf. If for all $\mu_1, \mu_2 \in I(\mathcal{L})$ and $\nu \in I(\sigma, \mathcal{L})$, such that if $\mu_1 \leq \mu_2, \nu(\mathcal{L})$ then $\mu_2 \leq \nu(\mathcal{L})$, it follows that $\mathcal{V}(\sigma, \mathcal{L})$ is slightly and mildly normal.

Proof. By Theorem 6.3, $\mathcal{V}(\sigma, \mathcal{L})$ is regular. As in the Corollary of Theorem 4.1, we show that $\mathcal{V}(\sigma, \mathcal{L})$ is Lindelöf and then it is also slightly and mildly normal. $\square$

Theorem 6.6. Consider $I(\sigma, \mathcal{L})$ and $\mathcal{V}(\sigma, \mathcal{L})$ which is weakly prime complete. Then $\mathcal{V}(\sigma, \mathcal{L})$ is prime complete if, and only if, for $\mu \in I_\sigma(\mathcal{L})$ there exists $\nu \in I(\sigma, \mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$.

Proof. Let $\mu \in I_\sigma(\mathcal{L})$ such that there exists $\nu \in I(\sigma, \mathcal{L})$ and $\mu \leq \nu(\mathcal{L})$. Consider the associated induced measures

$$\underline{\mu} \in I_\sigma(\mathcal{V}(\sigma, \mathcal{L})) \text{ and } \underline{\nu} \in I(\sigma, \mathcal{V}(\sigma, \mathcal{L})) \text{ for which } \underline{\mu} \leq \underline{\nu} \text{ on } \mathcal{V}(\sigma, \mathcal{L}).$$

$\mathcal{V}(\sigma, \mathcal{L})$ weakly prime complete implies $S(\underline{\nu}) \neq \emptyset$ and since $\underline{\mu} \leq \underline{\nu}$ it follows $S(\underline{\mu}) \neq \emptyset$, so that $\mathcal{V}(\sigma, \mathcal{L})$ is prime complete.

Conversely, suppose $\mathcal{V}(\sigma, \mathcal{L})$ is prime complete and let $\mu \in I_\sigma(\mathcal{L})$. Consider the corresponding $\underline{\mu} \in I_\sigma(\mathcal{V}(\sigma, \mathcal{L}))$ for which $S(\underline{\mu}) \neq \emptyset$, so there exists $\nu \in S(\underline{\mu})$ and $\mu \leq \nu$ on $\mathcal{V}(\sigma, \mathcal{L})$. Hence $\nu \in I(\sigma, \mathcal{L})$. $\square$

We can finally return to $I_R^\sigma(\mathcal{L})$ and the lattice $\mathcal{W}_\sigma(\mathcal{L})$, where $\mathcal{L}$ is disjunctive, and investigate when $\mathcal{W}_\sigma(\mathcal{L})$ is weakly prime complete.

Theorem 6.7. Let $\mathcal{L}$ be a disjunctive lattice and consider $I_R^\sigma(\mathcal{L})$. $\mathcal{W}_\sigma(\mathcal{L})$ is weakly prime complete if, and only if, for $\mu \in I(\sigma, \mathcal{L})$ there exists $\nu \in I_R^\sigma(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$.

Proof. Let $\mu \in I(\sigma, \mathcal{L})$ and consider the associated induced measure $\underline{\mu} \in I(\sigma, \mathcal{W}_\sigma(\mathcal{L}))$ defined by $\underline{\mu}(W_\sigma(L)) = \mu(L)$, $L \in \mathcal{L}$. $\mathcal{W}_\sigma(\mathcal{L})$ prime complete implies $S(\underline{\mu}) \neq \emptyset$, so that there exists $\nu \in S(\underline{\mu})$ and $\mu \leq \nu(\mathcal{L})$, $\nu \in I_R^\sigma(\mathcal{L})$.

Conversely, let $\underline{\mu} \in I(\sigma, \mathcal{W}_\sigma(\mathcal{L}))$ and consider the associated $\mu \in I(\sigma, \mathcal{L})$ such that $\underline{\mu}(W_\sigma(L)) = \mu(L)$, $L \in \mathcal{L}$. Assume now that there exists $\nu \in I_R^\sigma(\mathcal{L})$ such that $\mu \leq \nu(\mathcal{L})$. Then $\underline{\nu} \in I_R^\sigma(\mathcal{W}_\sigma(\mathcal{L}))$ and $\underline{\mu} \leq \underline{\nu}(W_\sigma(\mathcal{L}))$. Hence $S(\underline{\nu}) \subset S(\underline{\mu})$ and since $\mathcal{W}_\sigma(\mathcal{L})$ is replete, $S(\underline{\nu}) \neq \emptyset$. $\square$

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Résumé substantiel en français. Soit X un ensemble non vide quelconque, $\mathcal{L}$ un treillis de parties de X avec $\emptyset, X \in \mathcal{L}$, et $\mathcal{A}(\mathcal{L})$ l'algèbre engendrée par $\mathcal{L}$.

Définition 1: terminologie par rapport aux treillis. On dit que $\mathcal{L}$ est *disjonctif* si, pour $x \in X$ et $L_1 \in \mathcal{L}$ avec $x \notin L_1$, il existe $L_2 \in \mathcal{L}$ tel que $x \in L_2$ et $L_1 \cap L_2 = \emptyset$. On dit que $\mathcal{L}$ *sépare* les points de X si pour tous points $x, y \in X$ avec $x \neq y$, il existe $L \in \mathcal{L}$ tel que $x \in L$ et $y \notin L$. On dit que $\mathcal{L}$ est T_2 si pour $x, y \in X$ avec $x \neq y$, il existe $L_1, L_2 \in \mathcal{L}$ tels que $x \in L'_1, y \in L'_2$ et $L'_1 \cap L'_2 = \emptyset$.

Définition 2: terminologie par rapport aux mesures. $I(\mathcal{L})$ représente l'ensemble des mesures non triviales finiment additives sur $\mathcal{A}(\mathcal{L})$, à valeurs dans $\{0, 1\}$. On note $I_R(\mathcal{L})$ la classe des éléments de $I(\mathcal{L})$ qui sont $\mathcal{L}$ -réguliers, $I_\sigma(\mathcal{L})$ ceux qui sont σ -lisses sur $\mathcal{L}$, $I^\sigma(\mathcal{L})$ ceux qui sont σ -lisses sur $\mathcal{A}(\mathcal{L})$ et $I_R^\sigma(\mathcal{L})$ ceux de $I^\sigma(\mathcal{L})$ qui sont $\mathcal{L}$ -réguliers.

Définition 3: correspondance entre filtres et mesures. Un ensemble $\mathcal{F} \subset \mathcal{L}$ est un *filtre* lorsque i) $\emptyset \notin \mathcal{F}$, ii) $\mathcal{L}$ est stable pour les intersections finies et iii) $A \in \mathcal{F}$, $B \in \mathcal{L}$ et $A \subset B$, impliquent $B \in \mathcal{F}$. Nous résumons quelques résultats connus sur les correspondances entre les mesures à valeurs dans $\{0, 1\}$ et les filtres (voir [7]), et nous y référons à travers l'article.

Définition 4: propriétés topologiques des treillis. Le support de $\mu \in I(\mathcal{L})$, noté $S(\mu)$, est défini comme étant l'ensemble $\bigcap \{L \in \mathcal{L} \mid \mu(L) = 1\}$. On dit que $\mathcal{L}$ est *compact* si $S(\mu) \neq \emptyset$ pour toute mesure $\mu \in I_R(\mathcal{L})$. On dit que $\mathcal{L}$ est *dénombrablement compact* si $I_R(\mathcal{L}) = I_R^\sigma(\mathcal{L})$. On dit que $\mathcal{L}$ est *normal* si, pour toute mesure $\mu \in I(\mathcal{L})$, il existe une unique mesure $\nu \in I_R(\mathcal{L})$ telle que $\mu \leq \nu(\mathcal{L})$. On dit que $\mathcal{L}$ est *régulier* si pour toutes mesures $\mu_1, \mu_2 \in I(\mathcal{L})$ telles que $\mu_1 \leq \mu_2(\mathcal{L})$, on a $S(\mu_1) = S(\mu_2)$. On dit que $\mathcal{L}$ est *replet* si $S(\mu) \neq \emptyset$ pour toute mesure $\mu \in I_R^\sigma(\mathcal{L})$. On dit que $\mathcal{L}$ est *premier complet* si $S(\mu) \neq \emptyset$ pour toute mesure $\mu \in I_\sigma(\mathcal{L})$.

Définition 5: espace général de Wallman et topologie de Wallman. $I_R^\sigma(\mathcal{L})$ est muni de la *topologie de Wallman* en prenant comme base pour les fermés, les ensembles $W_\sigma(L) = \{\mu \in I_R^\sigma(\mathcal{L}) \mid \mu(L) = 1\}$ pour tout $L \in \mathcal{L}$. Dans ce cas on dit que $I_R^\sigma(\mathcal{L})$ est l'*espace général de Wallman* associé à X et $\mathcal{L}$. De façon analogue, on peut construire les espaces topologiques de Wallman $[I_\sigma(\mathcal{L}), t\mathcal{V}_\sigma(\mathcal{L})]$, $[I_R(\mathcal{L}), t\mathcal{W}(\mathcal{L})]$, et $[I(\mathcal{L}), t\mathcal{V}(\mathcal{L})]$.

Définition 6: mesure induite. La *mesure induite* $\underline{\mu} \in I_R^\sigma(\mathcal{L})$ par $\mu \in I_R^\sigma(\mathcal{L})$ est définie par $\underline{\mu}(W_\sigma(\mathcal{L})) = \mu(A)$ pour tout $A \in \mathcal{A}(\mathcal{L})$.

Théorème 3.1. (a) Considérons $I_R^\sigma(\mathcal{L})$ et $\mathcal{W}_\sigma(\mathcal{L})$ avec $\mathcal{L}$ disjonctif. Alors $\mathcal{W}_\sigma(\mathcal{L})$ est régulier si, et seulement si, pour tous $\mu_1, \mu_2 \in I(\mathcal{L})$ et $\nu \in I_R^\sigma(\mathcal{L})$ avec $\mu_1 \leq \mu_2, \nu(\mathcal{L})$, on a $\mu_2 \leq \nu(\mathcal{L})$.

(b) L'espace $[I_R^\sigma(\mathcal{L}), t\mathcal{W}_\sigma(\mathcal{L})]$ avec $\mathcal{L}$ disjonctif est T_2 si, et seulement si, $\mu \in I(\mathcal{L})$ et $\nu_1, \nu_2 \in I_R^\sigma(\mathcal{L})$ tels que $\mu \leq \nu_1, \nu_2(\mathcal{L})$ impliquent $\nu_1 = \nu_2$.

Théorème 3.2. $\mathcal{V}_\sigma(\mathcal{L})$ est régulier si, et seulement si, $I_\sigma(\mathcal{L}) = I_R^\sigma(\mathcal{L})$.

Définition 7. (a) $\mathcal{L}$ est à *peine normal* si pour tout $\mu \in I_\sigma(\mathcal{L}')$, il existe un unique $\nu \in I_R(\mathcal{L})$ tel que $\mu \leq \nu(\mathcal{L})$.

(b) $\mathcal{L}$ est *modérément normal* si pour tout $\mu \in I_\sigma(\mathcal{L})$, il existe un unique $\nu \in I_R(\mathcal{L})$ tel que $\mu \leq \nu(\mathcal{L})$.

(c) $\mathcal{L}$ est *presque dénombrablement compact* si $\mu \in I_R(\mathcal{L}')$ implique $\nu \in I_\sigma(\mathcal{L})$.

Théorème 4.1. Supposons que $\mathcal{L}$ est disjonctif.

(a) Considérons $I_R^\sigma(\mathcal{L})$ et $\mathcal{W}_\sigma(\mathcal{L})$ et supposons que $\mathcal{L}$ est Lindelöf et vérifie la condition : pour tous $\mu_1, \mu_2 \in I(\mathcal{L})$ et $\nu \in I_R^\sigma(\mathcal{L})$, si $\mu_1 \leq \mu_2, \nu(\mathcal{L})$ alors $\mu_2 \leq \nu(\mathcal{L})$. Dans ce cas, $\mathcal{W}_\sigma(\mathcal{L})$ est à *peine normal* et *modérément normal*.

(b) Si $\mathcal{L}$ est engendré par les compléments, alors $\mathcal{W}_\sigma(\mathcal{L})$ est à *peine normal*.

(c) Si $\mathcal{L}$ est presque dénombrablement compact et modérément normal, alors $\mathcal{W}_\sigma(\mathcal{L})$ est normal.

Nous considérons maintenant certaines conditions sur $\mathcal{L}$ et leurs conséquences.

Condition (1): Pour $\pi \in \mathcal{P}(\mathcal{L})$ quelconque, il existe $\nu \in I_\sigma(\mathcal{L})$ tel que $\pi \leq \nu(\mathcal{L})$.

Condition (2): Pour $\pi \in \mathcal{P}(\mathcal{L})$ quelconque, il existe $\nu \in I_R^\sigma(\mathcal{L})$ tel que $\pi \leq \nu(\mathcal{L})$.

Condition (3): Pour $\mu \in I_\sigma(\mathcal{L})$ quelconque, il existe $\nu \in I_R^\sigma(\mathcal{L})$ tel que $\pi \leq \nu(\mathcal{L})$.

Condition (4): Pour $\mu \in I_\sigma(\mathcal{L}')$ quelconque, il existe $\nu \in I_R^\sigma(\mathcal{L})$ tel que $\pi \leq \nu(\mathcal{L})$.

Théorème 4.2. (a) Si la condition (1) est vérifiée et si $\mathcal{L}$ est premier complet, alors $\mathcal{L}$ est Lindelöf.

(b) Si $\mathcal{L}$ est Lindelöf alors la condition (1) est vérifiée.

(c) $\mathcal{L}$ vérifie la condition (1) si, et seulement si, $[I_\sigma(\mathcal{L}), t\mathcal{V}_\sigma(\mathcal{L})]$ est Lindelöf.

Théorème 4.3. (a) Si $\mathcal{L}$ vérifie la condition (2) et est replet, alors $\mathcal{L}$ est Lindelöf.

(b) Si $\mathcal{L}$ est dénombrablement compact alors $\mathcal{L}$ vérifie la condition (2).

(c) Si $\mathcal{L}$ est disjonctif et Lindelöf alors $\mathcal{L}$ vérifie la condition (2).

(d) Si $\mathcal{L}$ est disjonctif alors $\mathcal{L}$ vérifie la condition (2) si, et seulement si, $[I_R^\sigma(\mathcal{L}), t\mathcal{W}_\sigma(\mathcal{L})]$ est Lindelöf.

Théorème 5.1. Soit $\mathcal{L}$ un treillis disjonctif. Alors $\mathcal{L}$ vérifie la condition (3) si, et seulement si, $\mathcal{W}_\sigma(\mathcal{L})$ est premier complet.

Théorème 5.2. (a) Soit $\mathcal{L}$ un treillis disjonctif, presque dénombrablement compact et modérément normal, et soit $\mathcal{W}_\sigma(\mathcal{L})$ premier complet. Alors $\mathcal{L}$ est dénombrablement compact.

(b) Soit $\mathcal{L}$ un treillis disjonctif, régulier, Lindelöf, et presque dénombrablement compact, et soit $\mathcal{W}_\sigma(\mathcal{L})$ premier complet. Alors $\mathcal{L}$ est dénombrablement compact.

Théorème 5.3. Soit $[I_\sigma(\mathcal{L}), t\mathcal{V}_\sigma(\mathcal{L})]$ un espace T_1 , $\mathcal{L}$ disjonctif et $\mathcal{W}_\sigma(\mathcal{L})$ premier complet. Alors $I_\sigma(\mathcal{L}) = I_R^\sigma(\mathcal{L})$.

Théorème 5.4. Soit $\mathcal{L}$ disjonctif.

(a) Si $\mathcal{L}$ vérifie la condition (4) alors $\mathcal{W}_\sigma(\mathcal{L})'$ est premier complet.

(b) Si $\mathcal{W}_\sigma(\mathcal{L})'$ est premier complet alors $\mathcal{L}'$ vérifie la condition (3).

Définition 8: fortement σ -lisse et faiblement premier complet. Une mesure $\mu \in I(\mathcal{L})$ est fortement σ -lisse sur $\mathcal{L}$ si, pour toute suite non décroissante $\{L_n\}$ avec $L_n \in \mathcal{L}$, si $\bigcap L_n \in \mathcal{L}$ alors $\mu(\bigcap L_n) = \inf_n \mu(L_n) = \lim_n \mu(L_n)$. On note $I(\sigma, \mathcal{L})$ la classe des mesures non triviales sur $\mathcal{L}$, à valeurs dans $\{0, 1\}$, qui sont fortement σ -lisses. On dit que $\mathcal{L}$ est faiblement premier complet si, pour $\mu \in I(\sigma, \mathcal{L})$, on a $S(\mu) \neq \emptyset$. On peut maintenant définir un autre espace topologique de Wallman $[I(\sigma, \mathcal{L}), t\mathcal{V}(\sigma, \mathcal{L})]$ et définir une condition semblable à la condition (1).

Condition (5): Pour $\pi \in \mathcal{P}_\sigma(\mathcal{L})$ quelconque, il existe $\nu \in I(\sigma, \mathcal{L})$ tel que $\pi \leq \nu(\mathcal{L})$.

Théorème 6.1. (a) Si la condition (5) est vérifiée et si $\mathcal{L}$ est faiblement premier complet, alors $\mathcal{L}$ est Lindelöf.

(b) Si $\mathcal{L}$ est Lindelöf alors la condition (5) est vérifiée.

Théorème 6.2. $\mathcal{L}$ vérifie la condition (5) si, et seulement si, $[I(\sigma, \mathcal{L}), t\mathcal{V}(\sigma, \mathcal{L})]$ est Lindelöf.

REFERENCES

1. A. D. Alexandroff, *Additive set functions in abstract spaces*, Math Sb (N.S.) 8 **50** (1940), 307–348.
2. G. Bachman and P. Stratigos, *Lattice repleteness and some of its applications to topology*, Journal of Mathematical Analysis and Applications **99** (1984), 472–492.
3. G. Bachman and M. Szeto, *On strongly measure replete lattices and the general Wallman remainder*, Fund.Math. **122** (1984), 199–217.
4. G. Bachman and A. Sultan, *Regular lattice measures: mappings and spaces*, Pacific J. Math. **67** (1976), 291–321.
5. R. Connell, *On certain Wallman spaces*, Internat. Journal Math. & Math. Sci. **17** (1994), 273–276.
6. G. Eid, *On normal lattices and Wallman spaces*, Internat. Journal Math. & Math. Sci. **13** (1990), 31–38.
7. Z. Frolik, *Prime filters with countable intersection property*, Comm. Math. Univ. Carolina **13** (1972), 553–575.
8. P. Grassi, *Outer measures and associated lattice properties*, Internat. Journal Math. & Math. Sci. **10** (1993), 687–694.
9. M. Munroe, *Measure and integration*, Addison-Wesley, 1971.
10. Pao-Sheng Hsu, *Applications of outer measures to separation properties of lattices and regular or σ -smooth measures*, Journal Math. & Math. Sci. **19** (1996), 253–262.
11. J. Ponnley, *Outer measures, measurability, and lattice regular measures*, Journal Math. & Math. Sci. **19** (1996), 343–350.
12. M. Szeto, *Measure repleteness and mapping preservations*, Journal Indian Math. Soc. **43** (1979), 35–52.
13. C. Vlad, *On lattice-topological properties of general Wallman spaces*, Internat. Journal Math. & Math. Sci. **21** (1998), 25–32.
14. C. Vlad, *Lattice separation and properties of Wallman type spaces*, Annali di Matematica pura ed applicata (IV) **CLV** (1991), 65–79.
15. H. Wallman, *Lattices and topological spaces*, Annals of Math. **39** (1938), 112–126.

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