

A NEW PROOF OF A THEOREM OF GREEN, HAPPEL AND ZACHARIA

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RÉSUMÉ. Nous donnons une nouvelle démonstration d'un théorème de Green, Happel et Zacharia pour calculer la dimension globale d'une algèbre triangulaire, monomiale, sobre, connexe et de dimension finie.

ABSTRACT. We give a new proof of a theorem of Green, Happel and Zacharia for computing the global dimension of a monomial triangular basic and connected finite dimensional algebra.

1. Introduction. It is a classical problem of algebra to find computational techniques for the homological invariants of a finite dimensional algebra (over a field, associative and with an identity). It was natural to try to apply the new methods of the representation theory of algebras as developed since the early seventies, to compute these invariants. One of the first works in this direction is due to E. L. Green, D. Happel and D. Zacharia [6]. They obtained an algorithm allowing to compute the minimal projective resolutions of the simple modules over a monomial algebra, and hence the global dimension of the latter.

Our objective here is to give a new proof of this result. While the proof in [6] consists in lifting the problem to the universal covering (which is a tree algebra), reducing the problem to computing the minimal projective resolutions of the simple modules over a Nakayama algebra, then calculating directly the latter, our proof uses formulae for the extension groups between simple modules [2]. We restrict ourselves to the case of triangular monomial algebras. The general case can be done by extending our results, using a covering that is triangular. The advantage of this approach is that it can be applied to not necessarily monomial algebras.

We use freely, and without further reference, concepts of homological algebra, as developed in [3] and of the representation theory of algebras, for which we refer to [1] and [5].

2. Preliminaries.

2.1. Notation. Let K be a commutative field and $Q = (Q_0, Q_1, s, t)$ be a finite connected and acyclic quiver (that is, having no oriented cycles of arrows). If $\alpha \in Q_1$ is an arrow, its source and target are denoted respectively by $s(\alpha)$ and $t(\alpha)$. A path w from i to j , of length n , is a composition of n arrows $w = \alpha_1 \dots \alpha_n$ such that $s(\alpha_1) = i$,

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$t(\alpha_n) = j$ and $t(\alpha_i) = s(\alpha_{i+1})$ for $1 \leq i < n$. If there exists such a path, we write $i \leq j$. Also we write $s(w) = i$ and $t(w) = j$. A bound quiver (Q, I) consists of a quiver Q and an ideal I of the path algebra KQ generated by linear combinations (called relations) of length ≥ 2 , and the quotient $A = KQ/I$ is called a bound quiver algebra. Recall that A is said to be triangular whenever Q is acyclic. It is well-known that A is basic and connected and that, conversely, if K is algebraically closed, any basic and connected finite dimensional K -algebra is of this form. If there exists a set ρ of paths of length ≥ 2 such that $I = \langle \rho \rangle$, we say that A is monomial. Throughout, our algebras are monomial bound quiver algebras with acyclic quiver, and all our modules are finitely generated right modules. We denote respectively by $S(i)$ and $P(i)$ the simple and the indecomposable projective modules corresponding to a point $i \in Q_0$.

We denote by $\text{gl. dim } A$ the global dimension of A . If $\Lambda = A/\text{rad } A = \bigoplus_{i \in Q_0} S(i)$, it is well-known that $\text{gl. dim } A = \sup \{n \mid \text{Ext}_A^n(\Lambda, \Lambda) \neq 0\} = \sup \{n \mid \text{there exist } i, j \in Q_0 \text{ and } \text{Ext}_A^n(S(i), S(j)) \neq 0\}$ (see [4] (11.4.6)).

2.2. The Ext-formulae. Let $A = KQ/I$ be a bound quiver algebra (not necessarily monomial), R be the ideal of KQ generated by the arrows of Q and $D = \text{Hom}_K(-, K)$ the duality functor. We have by [2] (1.1),

$$\text{Ext}_A^{2n}(\Lambda, \Lambda) \cong D \left[\frac{I^n \cap R I^{n-1} R}{R I^n + I^n R} \right] \quad \text{for } n \geq 1;$$

$$\text{Ext}_A^{2n+1}(\Lambda, \Lambda) \cong D \left[\frac{R I^n \cap I^n R}{I^{n+1} + R I^n R} \right] \quad \text{for } n \geq 0.$$

We deduce easily that, if e_i is the idempotent corresponding to $i \in Q_0$,

$$\text{Ext}_A^{2n}(S(i), S(j)) \cong D \left[e_i \left(\frac{I^n \cap R I^{n-1} R}{R I^n + I^n R} \right) e_j \right] \quad \text{for } n \geq 1;$$

$$\text{Ext}_A^{2n+1}(S(i), S(j)) \cong D \left[e_i \left(\frac{R I^n \cap I^n R}{I^{n+1} + R I^n R} \right) e_j \right] \quad \text{for } n \geq 0.$$

Finally, we recall that, if $\cdots \rightarrow P_m \rightarrow \cdots \rightarrow P_1 \rightarrow P(i) \rightarrow S(i) \rightarrow 0$ is a minimal projective resolution of $S(i)$, then $\text{Ext}_A^m(S(i), S(j)) \neq 0$ if and only if $P(j)$ is a direct summand of P_m .

3. Sequences of relations.

3.1. Let $A = KQ/I$ be a monomial algebra, where $I = \langle \rho \rangle$. We may, without loss of generality, assume that ρ is minimal in the sense that no monomial in ρ is a proper factor of any other monomial in ρ . For a given path w , we denote by $\rho|_w$ the set of monomials in ρ which are also monomial factors of w .

We define the *sequence of relations* $\mathcal{S}_w$ associated to the path w as follows. Let $\rho' = \rho|_w$ and suppose $s(w) = i$, $t(w) = j$. If there is no $r \in \rho'$ such that $s(r) = i$, let $\mathcal{S}_w = \emptyset$. Suppose there exists $r_1 \in \rho'$ such that $s(r_1) = i$, and let $R_1 = \{r_1\}$. Fix $R_2 = \{r \in \rho' \mid s(r_1) < s(r) < t(r_1)\}$. If $R_2 \neq \emptyset$, let $r_2 \in R_2$ be the path such that $s(r_2)$ is minimal in the set $\{s(r) \mid r \in R_2\}$. Inductively, suppose we have constructed $r_1, r_2, \dots, r_{k-1}$ with $k > 2$. Let $R_k = \{r \in \rho' \mid t(r_{k-2}) < s(r) \leq t(r_{k-1})\}$. If $R_k \neq \emptyset$, let $r_k \in R_k$ be the path such that $s(r_k)$ is minimal in the set $\{s(r) \mid r \in R_k\}$. If there exists N such that $t(r_N) = j$, so that $R_N = \{r_N\}$, then $\mathcal{S}_w = (r_1, \dots, r_N)$ is

the sequence of relations associated to w . We say that $\mathcal{S}_w$ is of *length* N and write $\ell(\mathcal{S}_w) = N$. If there is no k such that $t(r_k) = j$, then $\mathcal{S}_w = \emptyset$.

We agree that if $\mathcal{S}_w = \emptyset$, then $\ell(\mathcal{S}_w) = 0$. For $i \in Q_0$, let w_i be a path in Q such that $s(w_i) = i$ and $\ell(\mathcal{S}_{w_i})$ is maximal in the set of lengths of paths beginning at i . The path w_i is not uniquely determined but we only need the length $\ell(\mathcal{S}_{w_i})$ which is unique.

Let w be a path in Q and $\rho' = \rho|_w$. Two paths $a, b \in \rho'$ are said to *overlap* each other if $s(a) < s(b) < t(a)$ or if $s(b) < s(a) < t(b)$. We observe that, if $w \in I^n$, there exist n relations on w which do not overlap each other; this indeed comes from the definition of I^n . Also, in the sequence of relations $\mathcal{S}_w$, two relations overlap each other if and only if they are consecutive.

Lemma 3.2. *Let $\mathcal{S}_w = (r_1, \dots, r_N)$ be a sequence of relations associated to the path w . If $u \in I^n$ is a subpath of w , there exist n non-consecutive relations $r_{k_1}, \dots, r_{k_n}$ on u .*

Proof. As observed above, there exist n relations $a_1, \dots, a_n \in \rho$ on u which do not overlap each other. Since $s(a_p), t(a_p)$ are on u for $1 \leq p \leq n$, it is clear that for every p , there exists k (with $1 \leq k \leq N$) such that $a_p \in R_k$. Moreover, if $p \neq q$, the relations a_p, a_q cannot belong to the same R_k . Suppose, for $p \neq q$ that $a_p \in R_{k_p}, a_q \in R_{k_q}$ and $s(a_p) < s(a_q)$. Then clearly, $s(r_{k_p}) \leq s(a_p) < s(r_{k_q})$. There remains to show that the relations r_{k_p}, r_{k_q} do not overlap each other. If this were the case, then $k_q = k_p + 1$ (the only relations to overlap each other are the consecutive ones). Then $R_{k_q} = R_{k_p+1}$ and if $r \in R_{k_q}$ we have $s(r) < t(r_{k_q-1}) = t(r_{k_p})$. We obtain $s(a_p) < s(a_q) < t(r_{k_p}) \leq t(a_p)$, contradicting the fact that a_p and a_q do not overlap each other. $\square$

4. The main result.

Proposition 4.1. *Let $\mathcal{S}_w = (r_1, \dots, r_N)$ be a sequence of relations associated to a path w .*

- (1) *If N is even, then $w \in \left(I^{\frac{N}{2}} R \cap R I^{\frac{N}{2}}\right)$, but $w \notin \left(I^{\frac{N}{2}+1} + R I^{\frac{N}{2}} R\right)$.*
- (2) *If N is odd, then $w \in \left(I^{\frac{N+1}{2}} \cap R I^{\frac{N-1}{2}} R\right)$, but $w \notin \left(R I^{\frac{N+1}{2}} + I^{\frac{N+1}{2}} R\right)$.*

Proof. We only prove (1), the proof of (2) being similar. Let u_k be the subpath of w such that $s(u_k) = t(r_k)$ and $t(u_k) = s(r_{k+2})$ for $1 \leq k \leq N-2$ and u_k equals the stationary path at $t(r_k)$ for $k = N-1, N$. Let u', u'' be the subpaths of w such that $s(u') = s(r_1), t(u') = s(r_2), s(u'') = t(r_{N-1})$ and $t(u'') = t(r_N)$. Then clearly $u', u'' \in R$. Since N is even, then

$$w = r_1 u_1 r_3 u_3 \dots r_{N-1} u_{N-1} u'' = \left(\prod_{k=1}^{\frac{N}{2}} r_{2k-1} u_{2k-1} \right) u'' \in I^{\frac{N}{2}} R.$$

Similarly,

$$w = u' r_2 u_2 r_4 u_4 \dots r_N u_N = u' \left(\prod_{k=1}^{\frac{N}{2}} r_{2k} u_{2k} \right) \in R I^{\frac{N}{2}}.$$

Thus, we have $w \in (I^{\frac{N}{2}}R \cap RI^{\frac{N}{2}})$.

There remains to show that $w \notin (I^{\frac{N}{2}+1} + RI^{\frac{N}{2}}R)$. Since w is a path, it cannot be written as a sum of other paths. Thus it suffices to verify that $w \notin I^{\frac{N}{2}+1}$ and $w \notin RI^{\frac{N}{2}}R$. If $w \in I^{\frac{N}{2}+1}$, by (3.2), there exist $\frac{N}{2} + 1$ non-consecutive relations r_k on w . But now, there are N relations on w , hence at most $\frac{N}{2}$ non-consecutive ones. This contradiction yields $w \notin I^{\frac{N}{2}+1}$. Similarly, $w \notin RI^{\frac{N}{2}}R$. $\square$

Corollary 4.2. *If $\mathcal{S}_w$ is the sequence of relations associated to the path w such that $s(w) = i$, $t(w) = j$ and $\ell(\mathcal{S}_w) = N \geq 1$, then $\text{Ext}_A^{N+1}(S(i), S(j)) \neq 0$.*

Proposition 4.3. *Let $m \geq 2$.*

- (1) *If no relation starts at i , then $\text{Ext}_A^m(S(i), S(j)) = 0$.*
- (2) *If no relation ends at j , then $\text{Ext}_A^m(S(i), S(j)) = 0$.*

Proof. We only prove (1), the proof of (2) being similar. If no relation starts at the vertex i and if i is not a sink, then $\text{proj. dim. } S(i) = 1$. If i is a sink, then $\text{proj. dim. } S(i) = 0$. Thus $\text{Ext}_A^m(S(i), S(j)) = 0$ for $m \geq 2$. $\square$

Observe that, if there is no path between i and j , then trivially $\text{Ext}_A^m(S(i), S(j)) = 0$ for $m \geq 0$.

4.4. Let w be a path in Q , and consider w as a full and connected subquiver of Q . Then $A_w = Kw/\langle \rho|_w \rangle$ is a full and connected subcategory of A . We denote respectively by $S_w(i)$ and $P_w(i)$ the simple and indecomposable projective A_w -modules corresponding to a point i in the quiver w .

Lemma. *Let $m \geq 1$. Then $\text{Ext}_A^m(S(i), S(j)) \neq 0$ if and only if there exists a path w in Q from i to j such that $\text{Ext}_{A_w}^m(S_w(i), S_w(j)) \neq 0$.*

Proof. We assume that $m = 2n$ is even (the odd case being similar). We have

$$\text{Ext}_A^{2n}(S(i), S(j)) = D \left[e_i \left(\frac{I^n \cap RI^{n-1}R}{RI^n + I^n R} \right) e_j \right] \neq 0$$

if and only if there exists a path w from i to j , which is non-zero in $\frac{I^n \cap RI^{n-1}R}{RI^n + I^n R}$, that is, which belongs to $I^n \cap RI^{n-1}R$ but not to $RI^n + I^n R$. The latter condition is equivalent to $\text{Ext}_{A_w}^m(S_w(i), S_w(j)) \neq 0$. $\square$

Proposition 4.5. [6] (1.2). *Let B be a Nakayama algebra of quiver Q . If $i \in Q_0$, let $i+1 \in Q_0$ be its immediate successor (if it exists).*

- (1) *If i is a sink, then $0 \rightarrow P(i) \rightarrow S(i) \rightarrow 0$ is a minimal projective resolution of $S(i)$.*
- (2) *If i is not a sink and no relation starts at i , then $0 \rightarrow P(i+1) \rightarrow P(i) \rightarrow S(i) \rightarrow 0$ is a minimal projective resolution of $S(i)$.*
- (3) *If i is not a sink and $\mathcal{S}_{w_i} = (r_1, \dots, r_N)$ is not empty, then $0 \rightarrow P(x_N) \rightarrow \dots \rightarrow P(x_2) \rightarrow P(x_1) \rightarrow P(i+1) \rightarrow P(i) \rightarrow S(i) \rightarrow 0$ is a minimal projective resolution of $S(i)$, where $x_k = t(r_k)$ for $1 \leq k \leq N$.*

Proof. (1) and (2) being trivial, we only prove (3). Let

$$\cdots \rightarrow P_n \rightarrow \cdots \rightarrow P_2 \rightarrow P_1 \rightarrow P(i) \rightarrow S(i) \rightarrow 0$$

be a minimal projective resolution of $S(i)$. Since i is not a sink, $P_1 = P(i+1)$. By (4.2), we have $\text{Ext}_B^{k+1}(S(i), S(x_k)) \neq 0$, therefore $P(x_k)$ is a direct summand of P_{k+1} . Since B is a Nakayama algebra, each P_k is indecomposable, hence $P(x_k) = P_{k+1}$ so that our minimal projective resolution becomes

$$\cdots P_{N+2} \rightarrow P(x_N) \rightarrow P(x_{N-1}) \rightarrow \cdots \rightarrow P(x_1) \rightarrow P(i+1) \rightarrow P(i) \rightarrow S(i) \rightarrow 0.$$

There remains to show that the morphism $P(x_N) \rightarrow P(x_{N-1})$ is injective. By definition of $\mathcal{S}_{w_i}$, there exists no relation $a \in \rho$ such that $t(r_{N-1}) \leq s(a) < t(x_N)$. Consequently $P(x_N) \subseteq P(x_{N-1})$. Since, on the other hand, there exists only one path from x_{N-1} to x_N , we have $\dim_K \text{Hom}_B(P(x_N), P(x_{N-1})) = 1$. Our claim follows. $\square$

Corollary 4.6. *If $i, j \in Q_0$ are not connected by any sequence of relations, then $\text{Ext}_A^m(S(i), S(j)) = 0$ for $m \geq 2$.*

Proof. It suffices by (4.4) to prove the statement in case A is a Nakayama algebra, whose quiver is a path from i to j , and by (4.3), to consider the case where there is a relation starting at i . Let $\mathcal{S}_{w_i} = (r_1, \dots, r_N)$ be a sequence of relations beginning at i and of maximal length. Since i and j are not connected by a sequence of relations, we have $j \neq t(r_k) = x_k$ for any $1 \leq k \leq N$. By (4.5) we have a minimal projective resolution

$$0 \rightarrow P(x_N) \rightarrow \cdots \rightarrow P(x_1) \rightarrow P(i+1) \rightarrow P(i) \rightarrow S(i) \rightarrow 0.$$

Now $\text{Ext}_A^m(S(i), S(j)) \neq 0$ for some $m \geq 2$ implies $P(j) = P(x_k)$ for some k : an absurdity since $j \neq x_k$ for all k . $\square$

Proposition 4.7. *If $i, j \in Q_0$ and $m \geq 2$, then $\text{Ext}_A^m(S(i), S(j)) \neq 0$ if and only if there exists a sequence of relations $\mathcal{S}_w = (r_1, \dots, r_{m-1})$ associated to a path w from i to j .*

Proof. By (4.6), $\text{Ext}_A^m(S(i), S(j)) \neq 0$ implies that i, j are connected by a sequence of relations associated to a path w such that $s(w) = i, t(w) = j$. Let $\mathcal{S}_w = (r_1, \dots, r_N)$ and $x_k = t(r_k)$. Since $\text{Ext}_A^m(S(i), S(j)) \neq 0$, then $\text{Ext}_{A_w}^m(S_w(i), S_w(j)) \neq 0$. Since $x_N = j$, then

$$0 \rightarrow P_w(j) \rightarrow P_w(x_{N-1}) \rightarrow \cdots \rightarrow P_w(x_1) \rightarrow P_w(i+1) \rightarrow P_w(i) \rightarrow S_w(i) \rightarrow 0$$

is a minimal projective resolution of A_w -modules. In particular, $P_w(j) = P_w(x_N)$ and $\text{Ext}_{A_w}^m(S_w(i), S_w(j)) \neq 0$ gives $N = m - 1$. The converse follows from (4.2). $\square$

Theorem 4.8. *If $A = KQ/\langle \rho \rangle$ is a monomial triangular non hereditary algebra, then*

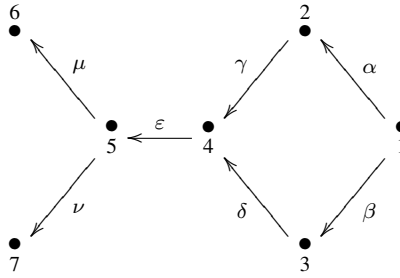
$$\text{gl. dim. } A = \sup \{ \ell(\mathcal{S}_{w_i}) + 1 \mid i \in Q_0 \text{ and there exists } r \in \rho \text{ with } s(r) = i \}.$$

Proof. Since A is not hereditary, $\rho \neq \emptyset$ and $\text{gl. dim } A \geq 2$. Then

$$\begin{aligned} \text{gl. dim } A &= \sup \{m \mid m \geq 2, \text{Ext}_A^m(S(i), S(j)) \neq 0 \text{ for } i, j \in Q_0\} \\ &= \sup \{m \mid m \geq 2, \text{there exists a path } w \text{ in } Q \text{ such that } s(w) = i, \\ &\quad t(w) = j, \mathcal{S}_w \neq \emptyset \text{ and } \ell(\mathcal{S}_w) + 1 = m\} \\ &= \sup \{\ell(\mathcal{S}_{w_i}) + 1 \mid i \in Q_0 \text{ and there exists } r \in \rho \text{ with } s(r) = i\}. \quad \square \end{aligned}$$

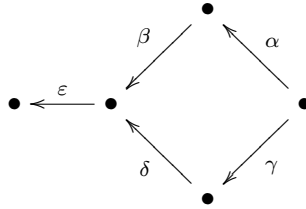
5. Examples.

Example 5.1. Let A be the monomial algebra given by the quiver



bound by $\rho = \{\alpha\gamma, \beta\delta, \gamma\varepsilon, \delta\varepsilon\nu, \varepsilon\mu\}$. Then $w_1 = \alpha\gamma\varepsilon\mu$, $w_2 = \gamma\varepsilon\mu$, $w_3 = \delta\varepsilon\nu$, $w_4 = \varepsilon\mu$ and $\ell(\mathcal{S}_{w_1}) = 3$, $\ell(\mathcal{S}_{w_2}) = 2$, $\ell(\mathcal{S}_{w_3}) = \ell(\mathcal{S}_{w_4}) = 1$, so that $\text{gl. dim } A = \ell(\mathcal{S}_{w_1}) + 1 = 4$.

Example 5.2. The Ext-formulae in (2.2) can be applied to not necessarily monomial algebras. Let A be given by the quiver



bound by $\rho = \{\alpha\beta - \gamma\delta, \beta\varepsilon\}$. Since $I = \langle \rho \rangle$, we have $I^2 = 0$ thus $\text{Ext}_A^n(\Lambda, \Lambda) = 0$ for all $n \geq 4$. Moreover $\text{Ext}_A^3(\Lambda, \Lambda) = D\left[\frac{RI \cap IR}{RIR}\right] = 0$ since $RI = \langle \alpha\beta\varepsilon \rangle$, $IR = \langle (\alpha\beta - \gamma\delta)\varepsilon \rangle$ and $RI \cap IR = 0$.

Finally, $\text{Ext}_A^2(\Lambda, \Lambda) = D\left[\frac{I \cap R^2}{RI + IR}\right] = D\left[\frac{I}{RI + IR}\right] \neq 0$ since $\beta\varepsilon \in I$ but $\beta\varepsilon \notin (RI + IR) \subseteq R^3$, so that $\text{gl. dim } A = 2$. Now if B is given by the same quiver as A , but is bound by $\rho' = \{\alpha\beta - \gamma\delta, \beta\varepsilon, \delta\varepsilon\}$, by the same argument, $\text{Ext}_B^n(\Lambda, \Lambda) = 0$ for $n \geq 4$. But $\text{Ext}_B^3(\Lambda, \Lambda) = D\left[\frac{RI \cap IR}{RIR}\right] \neq 0$, since $(\alpha\beta\varepsilon - \gamma\delta\varepsilon)$ belongs to $RI \cap IR$ but not to $RIR \subseteq R^4$, so that $\text{gl. dim } B = 3$. We recall that the case of incidence algebras of partially ordered sets was solved in [7].

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Résumé substantiel en français. L'objectif de cet article est de développer une méthode de calcul de la dimension globale d'une algèbre triangulaire monomiale. Nous considérons les algèbres A de la forme $A = KQ/I$ où K est un corps commutatif, Q un carquois fini connexe et acyclique et I un idéal de KQ engendré par des chemins de Q de longueur au moins 2. De telles algèbres sont dites triangulaires monomiales. Nous notons ρ l'ensemble des chemins qui engendrent I .

La dimension globale d'une algèbre A (notée $\dim . gl . A$) est par définition le plus grand entier m tel qu'il existe des A -modules M, N avec $\text{Ext}_A^m(M, N) \neq 0$. S'il n'existe aucun tel entier, alors $\dim . gl . A = \infty$. Il est connu que, si Λ est le A -module $A/\text{rad } A$, alors on peut remplacer $\text{Ext}_A^m(M, N)$ par $\text{Ext}_A^m(\Lambda, \Lambda)$ dans la définition précédente. Il existe une formule donnant ces extensions (voir [2] (1.1)) : si $D = \text{Hom}_K(-, K)$ est la dualité entre A -modules et A^{op} -modules, si $A = KQ/I$ et R est l'idéal de KQ engendré par les chemins non triviaux de Q , alors

$$\text{Ext}_A^{2n}(\Lambda, \Lambda) = D \left[\frac{I^n \cap R I^{n-1} R}{R I^n + I^n R} \right] \quad \text{pour } n \geq 1;$$

$$\text{Ext}_A^{2n+1}(\Lambda, \Lambda) = D \left[\frac{R I^n \cap I^n R}{I^{n+1} + R I^n R} \right] \quad \text{pour } n \geq 0.$$

Nous définissons une notion de suite de relations associée à un chemin de Q , laquelle nous permet de calculer pour un point i du carquois Q , une suite de relations $\mathcal{S}_{w_i}$ associée à un chemin w_i ayant i pour source et de longueur maximale. Nous notons cette longueur $\ell(\mathcal{S}_{w_i})$.

Nous pouvons maintenant énoncer le résultat principal de cet article. Si $A = KQ/\langle \rho \rangle$ est une algèbre triangulaire monomiale non héréditaire, alors

$$\dim . gl . A = \sup \{ \ell(\mathcal{S}_{w_i}) + 1 \mid i \in Q_0 \text{ et il existe } r \in \rho \text{ avec } s(r) = i \}$$

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